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Multiobjective Optimization of Structures with and Without Control

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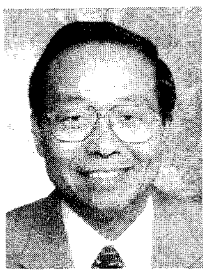
A multiobjective optimization algorithm is introduced, and the effect of different objective functions on structural design is investigated. The basic idea of this approach is to support the decisionmaker by directly selecting the best compromise solution from the Pareto optimal solutions. A payoff matrix is first constructed by optimizing each objective function subject to the given constraints; a substitute function is then formed. Under the original constraint conditions, maximizing the substitute function gives the best compromise solution. This research shows that multiobjective optimization should be carried out to obtain a rational structural design. Two structural optimal design examples—one frame structure and one frame structure with active control—illustrate the application of this approach. Numerical results show that the rational compromise solutions are achieved by multiobjective optimization procedure.

Introduction

DURING the past two decades, a number of methods have been developed that are capable of dealing with multiobjective optimization problems.[1–4] These approaches can be divided into two fundamental categories. First, the original multiobjective optimization problem is transformed into a scalar substitute one that contains contributions from all of the objectives. Weighting method,[5] goal programming,[6] game theory,[7] and other methods belong to this category. Second, vector optimization techniques are used. They include the minimax method,[8] ε-constrain method,[9] and sequential method.[10] The present work demonstrates a cooperative multiobjective optimization algorithm. Its advantage is to make each individual objective function obtain the maximum possible benefit in their tradeoff procedure so that the best compromise solution can be achieved from the Pareto optimal solutions. Analysis with the algorithm is composed of the formation of a substitute criterion and the scalar optimization computation. This method is easy to handle because it involves only the scalar optimization procedure; the substitute function can directly decide the compromise solution.

Structural optimization is basically a decision based on different design requirements. In the design process, a set of decision criteria, which will be used to evaluate the possible results, are imposed.[11–13] These criteria form the structural optimization problem. Most real-world optimization problems in structures are multimodal. There often exist several criteria to be considered by the designer. Usually these objectives are conflicting, or competing, rather than complementary. For these problems, a multiobjective formulation is appropriate. By investigating the effect of different objective functions on structural design and comparing the solutions from single and multiple objective structural optimization, it is observed that the multiobjective model provides a tool to find the tradeoff between conflicting objectives. Since multiobjective optimization effectively considers all of the different, mutually conflicting requirements inherent in a design problem, the final optimal design stands as a good compromise.

The objective of vibration control is to design a structure and its control systems to reduce structural dynamic response to a desired level. Traditionally, the structure and control systems have been designed separately. Structural engineers and control engineers have different design objectives. Obviously, the traditional approach cannot yield the best overall design. It is wise to use all design resources because of strong interaction between the structure and control



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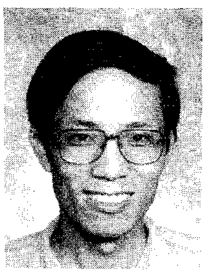

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systems. Therefore, simultaneous structure/control design is necessary to achieve ideal control with minimum cost. In recent years, a great deal of research effort has involved new design methods.[14–19] Research shows that an integrated design of the structure/control system is obtained by improving the configuration, as well as the control system. A real-world integrated structure/control optimal design is a multiobjective optimization problem; designers must deal with more than one objective to meet the optimization requirements of the structure and control systems. Until now, few researchers[14,17,20,21] treated the multiobjective optimization problem in the integrated design of structure and control system. In this paper, the design problem of structure with control is formulated as a multiobjective optimization problem. A numerical example is presented to demonstrate the analysis procedure.

Multiobjective Optimization Algorithm

Multiobjective optimization can be defined as determining a vector of design variables that is within the feasible region and minimizing (or maximizing) a vector of objective function. It can be expressed as follows.

Minimize:

$$F(x) = \{f_1(x), f_2(x), \ldots, f_m(x)\}$$

subject to

$$g(x) \leq 0 \tag{1}$$

where $g(x)$ stands for the constraint vector and $f_i(x)$ is the ith objective function.

The main feature of the preceding optimization problem is the appearance of an objective conflict, namely, that none of the feasible solutions allows simultaneously minimizing all objectives. Thus, the solution of a multiobjective optimization problem can be defined as follows. If vector x^* is a solution to Eq. (1), there exists no feasible vector x that would decrease some objective functions without causing a simultaneous increase in at least one objective function. The definition of the solution is the same as that of the Pareto optimum (nondominated solution).

A feasible vector x^* is a Pareto optimum for Eq. (1) if and only if there exists no feasible vector x such that[1] for all $i \in \{1, 2, \ldots, m\}$

$$f_i(x) \leq f_i(x^*)$$

and for at least one $i \in \{1, 2, \ldots, m\}$

$$f_i(x) < f_i(x^*)$$

Figure 1 shows a problem with two objective functions and two design variables where the Pareto optimal solution lies on curved section AB.

One of the multiobjective optimization algorithms reduces a vector optimization problem into a scalar substitute one. To accomplish this, a so-called substitute problem, which transforms the multiobjective optimization problem into a scalar optimization problem (single-objective optimization), is defined. There are various methods for transforming vector optimization problems into substitute problems.[1–4] The key to the transformation is that the solution $\bar{x}$ of the substitute problem should be a point in the Pareto set with respect to feasible region and set of objective functions. Among transformation methods is the weighting method, a scalar objective formulated as a weighted sum of individual objective functions; it is the most commonly used approach. Different Pareto optimal solutions can be generated by varying weights of the objective functions. If the weight of an individual objective function can be determined in advance, the weighting optimal method can be applied to a multiobjective optimization problem. If these weights cannot be determined in advance, a decision-making process that picks the best solution from the Pareto optimal set is applied. A number of publications have dealt with various methods for the decision-making process.[1,3,4,17]

Consider a two-objective optimization problem. Let $f_1(x)$ and $f_2(x)$ represent dual scalar objectives, and x the design variable vector. These objectives are in conflict. As shown in Fig. 2, point A is the best value for objective f_1 and point B the best for objective f_2. With cooperative multiobjective optimization, the best solution should guarantee that each objective obtains its maximum possible value, although each objective cannot achieve its own best value. Obviously, the ideal point is not point A or B; it is a point that lies on the line of AB (Pareto set). Whatever point is chosen by this principle represents the best compromise solution. To select the ideal point in Eq. (1), a solution procedure is constructed as follows.

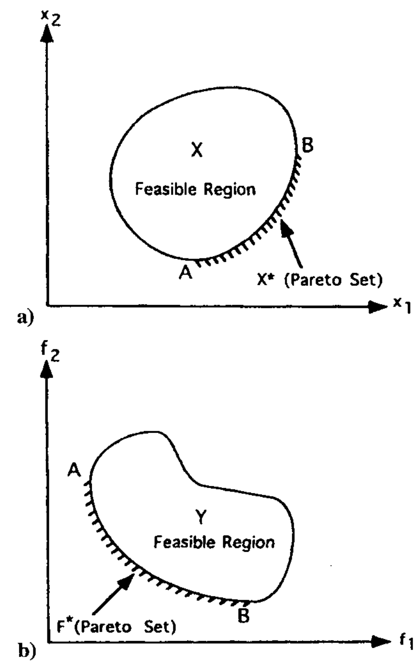

Fig. 1 Feasible region and Pareto set: a) in decision space and b) in objective space.

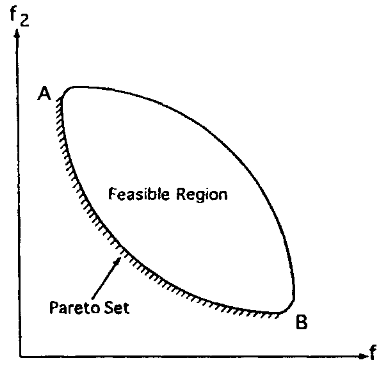

Fig. 2 Feasible region and Pareto set in objective space.

First, the m individual objective functions are minimized subject to the given constraints.

Minimize:

$$f_i(x)$$

subject to

$$g(x) \leq 0 \tag{2}$$

For each objective function f_i, an optimal solution x_1^* is obtained. Then a payoff matrix is constructed as

$$[P_0] = \begin{bmatrix} f_1(x_1^*), f_2(x_1^*), \ldots, f_m(x_1^*) \\ f_1(x_2^*), f_2(x_2^*), \ldots, f_m(x_2^*) \\ \cdots \quad \cdots \\ f_1(x_m^*), f_2(x_m^*), \ldots, f_m(x_m^*) \end{bmatrix} \tag{3}$$

For each objective function, its best and worst values in the Pareto set can be obtained from the preceding matrix:

$$\begin{aligned} f_{i,\min} &= f_i(x_i^*) \qquad i = 1, \ldots, m \\ f_{i,\max} &= \max\left[f_i(x_j^*)\right] \qquad j = 1, \ldots, m \qquad i = 1, \ldots, m \end{aligned} \tag{4}$$

From the payoff matrix, during cooperative optimization, the ith objective function should not expect a value better than $f_{i,\min}$, but the value should not be worse than $f_{i,\max}$. Based on this consideration, a substitute objective function can be constructed as

$$S = \prod_{i=1}^{m} \frac{[f_{i,\max} - f_i(\boldsymbol{x})]}{[f_{i,\max} - f_{i,\min}]} = \prod_{i=1}^{m} \bar{f}_i(\boldsymbol{x}) \tag{5}$$

Equation (5) shows that maximizing the substitute function S achieves the best compromise solution for the objective functions. When $S = 1$, all objectives achieve their best values (impossible in a well-defined multiobjective optimal problem). When $S = 0$, at least one of the objectives achieves its worst value, which cannot be accepted by the corresponding objective. The value range of the substitute function S should be that of $0 < S < 1$.

The multiobjective optimization of Eq. (1) becomes

Maximize:

$$S = \prod_{i=1}^{m} \bar{f}_i(x)$$

subject to

$$g(x) \leq 0 \tag{6}$$

As noted, the solution of a multiobjective optimization problem must come from the Pareto set.

Assume $\boldsymbol{x}^*$ is the solution of Eq. (5) but is not a Pareto optimal solution. Then there exists an $\bar{x}$ that belongs to the feasible zone, such that $f_j(\bar{x}) < f_j(\boldsymbol{x}^*)$, and for $i \neq j$, $f_j(\bar{x}) \leq f_j(\boldsymbol{x}^*)$, implying

$$\begin{aligned} S(\bar{x}) &= \prod_{i=1}^{m} \frac{[f_{i,\max} - f_i(\bar{x})]}{[f_{i,\max} - f_{i,\min}]} \\ &= \frac{[f_{j,\max} - f_j(\bar{x})]}{[f_{j,\max} - f_{j,\min}]} \prod_{\substack{i=1 \\ i \neq j}}^{m} \frac{[f_{i,\max} - f_i(\bar{x})]}{[f_{i,\max} - f_{i,\min}]} \\ &> \frac{[f_{j,\max} - f_j(\boldsymbol{x}^*)]}{[f_{j,\max} - f_{j,\min}]} \prod_{\substack{i=1 \\ i \neq j}}^{m} \frac{[f_{i,\max} - f_i(\boldsymbol{x}^*)]}{[f_{i,\max} - f_{i,\min}]} \\ &= \prod_{i=1}^{m} \frac{[f_{i,\max} - f_i(\boldsymbol{x}^*)]}{[f_{i,\max} - f_{i,\min}]} = S(\boldsymbol{x}^*) \end{aligned} \tag{7}$$

This contradicts the assumption that $\boldsymbol{x}^*$ is the optimal solution of Eq. (5) and so $\boldsymbol{x}^*$ must be a Pareto-optimal solution.

Therefore, the solution of Eq. (5) is the best solution as selected from the Pareto set of Eq. (1). The proposed multiobjective optimization algorithm needs $m + 1$ scalar optimization procedures. When the substitute function is constructed, it is only a single objective optimization problem.

Objective Functions and Constraints

In a structural system optimization problem, cross-sectional areas and moments of inertia of the structural members are chosen as design variables. Constraints include displacements, stresses, frequencies, and buckling loads, as well as upper and lower bounds of the design variables. Objective functions are constructed based on various optimization purposes.

To better utilize materials and reduce costs of a structure, structural weight can be chosen as the objective function:

$$f_1 = W = \sum \rho_i A_i l_i \tag{8}$$

where ρ_i, A_i, and l_i are weight density, cross-sectional area, and length of member i, respectively.

In a given structure, when the internal work (strain energy) done by stresses and strains has a minimum value, the structure has an optimal shape. For example, minimizing the strain energy of a truss structure produces a natural structural shape. Strain energy is given by

$$f_2 = E_s = \sum \sigma_i \varepsilon_i v_i \tag{9}$$

where σ_i, ε_i, and v_i are stress, strain, and volume of member i, respectively.

Minimizing potential structural energy can reduce the effects of external forces and increase the safety level of a structure. If the given loads are $\{\bar{P}\} = \{p_1, p_2, \ldots, p_n\}$, and the corresponding displacements are $\{\Delta\} = \{\delta_1, \delta_2, \ldots, \delta_n\}$, the potential energy of the structure is

$$f_3 = E_p = \{\bar{P}\}^T\{\Delta\} = \sum p_i \delta_i \tag{10}$$

To minimize changes in the shape of a structure under the action of different loading conditions, displacements at selected points or regions of the structure are taken as the objectives

$$f_{4,i} = \delta_i \tag{11}$$

When a structure is subjected to earthquake excitations, reducing dynamic response and damage caused by seismic loads is the main design consideration. Decreasing the dynamic response can be done by minimizing acceleration of masses, earthquake input energy, etc. Earthquake input energy of a structure represents the work done by the structure's base shear as it moves through ground displacement and can be expressed as (see Appendix)

$$f_5 = E_1 = \sum D_i^2 \left(\tfrac{1}{2}\mathrm{MS}_{vi}^2\right) \tag{12}$$

where $D_i = \{\phi_i\}^T\{m\}/\sqrt{M}$ is the ith mode energy parameter and $\{\phi_i\}$ the normalized ith mode. Also, $\{m\} = \{m_1, m_2, \ldots, m_n\}$, and $M = \sum m_i$ are the structure's total mass. Furthermore, maximizing the reliability and safety of a structure can be taken as the structural optimization objective for different optimal choices.[22]

For a control system optimization problem, design variables include feedback gain and passive parameters of a passive control system. Constraints are put on the closed-loop damping factor, frequencies, and design requirements. Usually, optimization of a control system minimizes a specified performance index for the purpose of reducing control energy or effective damping response time. To design a controller using a linear quadratic regulator, a performance index (PI) can be defined as[23]

$$PI = \int_0^{t_f} (\{z\}^T[Q]\{z\} + \{\boldsymbol{u}\}^T[R]\{\boldsymbol{u}\})\,\mathrm{d}t \tag{13}$$

where $[Q]$ is the positive semidefinite state weighting matrix, $[R]$ the positive definite control weighting matrix, $\{z\}$ the state displacement vector, and $\{\boldsymbol{u}\}$ the control input vector. Minimizing the quadratic performance index and satisfying the structural system state equation gives the state feedback control law

$$\{\boldsymbol{u}\} = -[R]^{-1}[B]^T[P]\{z\} = -[G]\{z\} \tag{14}$$

where $[G]$ is the closed-loop gain matrix and $[P]$ is the Riccati matrix. The objective function to minimize the performance index can be taken as[21]

$$f_6 = \{z_0\}^T[P]\{z_0\} \tag{15}$$

Here $\{z_0\}$ is the initial state vector (disturbance vector).

The effective damping response time is given by[24]

$$f_7 = \frac{\{z_0\}^T[P]\{z_0\}}{\{z_0\}^T[Q]\{z_0\}} \tag{16}$$

where $[Q]$ is the weighting matrix.

Obviously, a designer has to deal with more than one objective to meet the design requirements of a structure or a structure/control system in most real-world optimization problems. Therefore, when optimization is concerned with real structures, a multiobjective optimization problem is formulated.

Multiobjective Optimization of Structures

Considering a three-story steel shear frame, as shown in Fig. 3, floor diaphragms are rigid and axial deformations are neglected. Thus, the system has only one degree of freedom (in the lateral direction) at each floor.

The amount of live and dead load at each story is 56 kN/m, which does not include column weight. Lateral forces (F_1 = 34 kN, F_2 = 52 kN, and F_3 = 36 kN) are shown in Fig. 3. Weight density and elastic module of steel are ρ = 7800 kg/m^3 and E = 200 * 10^6 kN/m^2, respectively. Design variables comprise the column moment of inertias I_i. The mass m_i of the ith story can be calculated as

$$m_i = 56 * 6/g + 2 * 4 * A_i * \rho \qquad (17)$$

where A_i is the column cross-sectional area of the ith story and g is the acceleration of gravity. For an American Institute of Steel Construction standard wide-flange section, section properties can be expressed in terms of moment of inertia as[11]

$$A = 0.80 I^{\frac{1}{2}} \qquad S = 0.78 I^{\frac{3}{4}} \qquad (18)$$

where S is the section module.

Optimum constraint conditions are

$$|\sigma_i| \leq 165{,}000\,\mathrm{kN/m^2} \qquad |\delta_i| \leq \tfrac{1}{400} h_i \qquad T_1 \geq 0.3\,\mathrm{s}$$
$$0.00001 \leq I_i \leq 0.002\,\mathrm{m^4} \qquad 0.5 \leq k_{i+1}/k_i \leq 1 \qquad (19)$$

where σ_i, δ_i, h_i, and k_i are column stress, relative story displacement, story height, and story stiffness of the ith story, respectively, and $i = 1, 2, 3$. T_1 is the structure's fundamental natural period.

To assess the impact of scalar and multiobjective optimization, as well as different objectives on structural design, the following nine cases are analyzed: case 1, minimize W [weight, Eq. (8)]; case 2, minimize E_s [strain energy, Eq. (9)]; case 3, minimize E_p [potential energy, Eq. (10)]; case 4, minimize E_i [input energy, Eq. (12)]; case 5, minimize $\{W, E_s\}$; case 6, minimize $\{W, E_p\}$; case 7, minimize $\{W, E_i\}$; case 8, minimize $\{W, E_s, E_i\}$; and case 9, minimize $\{W, E_s, E_p, E_i\}$.

Solutions are presented in Table 1 and in Figs. 4–6 for these cases. Earthquake wave records used in this example are from El-Centro, N-S, 1940.

The following observations can be made from the results.

1) It may not be possible to produce a good compromise design in the context of a single objective optimization (see Figs. 4–6). For example, when a structure is optimized for minimum weight subject to imposed constraints, dynamic responses of the structure are strong. Otherwise, when optimization is applied to the earthquake input energy of the structure, small dynamic responses are accompanied by large structural weight. Multiobjective optimization can achieve a complementary solution from the conflict objectives.

2) From Table 1, it can be observed that single objective optimization often produces a design in which two constraint types are active, which means that the final design is bound by several constraint

Table 1 Active constraints

Case	1	2	3	4	5	6	7	8	9
$\lvert\sigma_i\rvert \leq \sigma_{ia}$									
$\lvert\delta_1\rvert \leq \delta_{1a}$	*								
$\lvert\delta_2\rvert \leq \delta_{2a}$	*								
$\lvert\delta_3\rvert \leq \delta_{3a}$									
$T_1 \geq T_a$		*	*	*					
$I_{il} \leq I_i \leq I_{iu}$									
$k_{i+1}/k_i \leq 1$							*		
$k_{i+1}/k_i \geq 0.5$	*	*	*	*	*	*		*	*

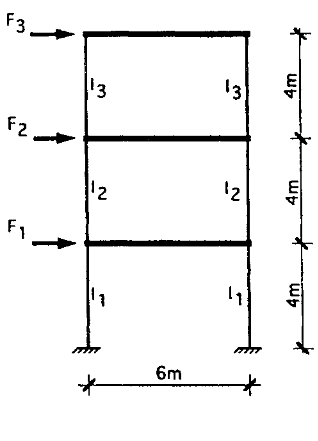

Fig. 3 Three-story steel shear frame.

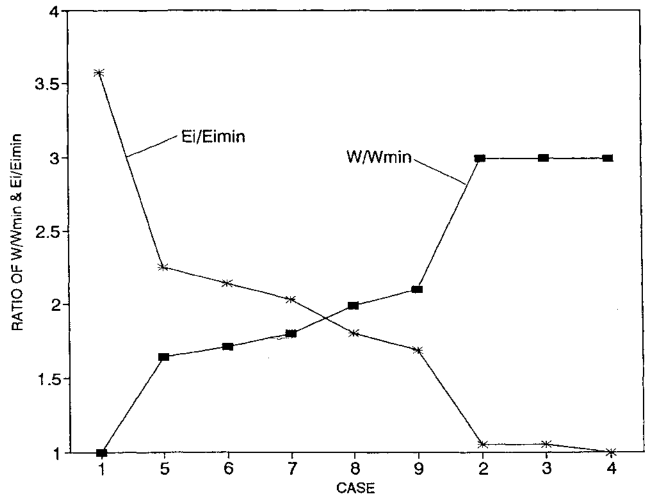

Fig. 4 Single and multiobjective optimization.

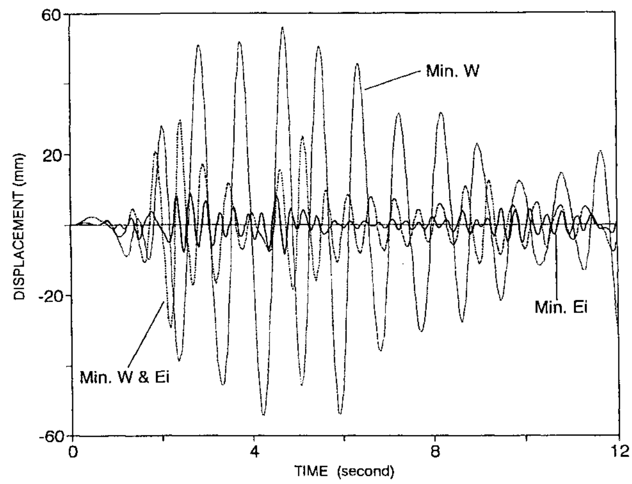

Fig. 5 Comparison of top-story displacement.

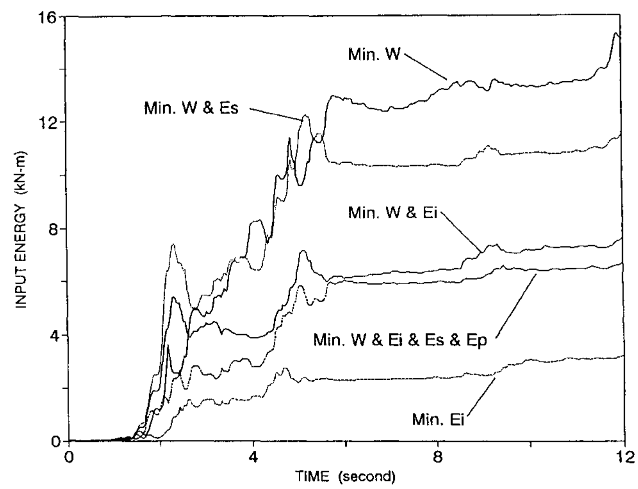

Fig. 6 Comparison of seismic input energy.

modes. Therefore, the solution may be sensitive to changes in constraints.

3) For single-objective structural optimization, if the physical characteristics of the members are fixed and the cross-sectional areas of members are taken as design variables, there are only two groups of objectives. One increases the cross-sectional areas, whereas the other decreases them. In multiobjective optimization, taking more objectives from one group means putting a greater weight on that group.

Multiobjective Optimization of Structures with Control

In the application of structure and control systems, designers hope to minimize costs and control efforts and simultaneously satisfy the desired structural requirements. Frequently, structural optimization seeks to minimize mass subject to various open-loop constraints (deflections, stresses, frequencies, etc.). The control system is optimized to minimize a specified performance index. Consider the situation for an integrated structure/control optimization design. Here, a multiobjective optimization problem should be formulated because designers have to deal with more than one objective to meet the optimization requirements of the structure and control systems. The solution procedure for integrated structure/control optimization design can be stated as follows. Cross-sectional areas of the structural members are taken as design variables. Constraints are applied to the structure (stresses, displacements, etc.) and the control requirements (closed-loop damping factors, frequencies, etc.). Objectives can come from the structural system, control system, or both, depending on design purpose.[25]

Concepts as just discussed are applied to the three-story steel frame structure investigated in the preceding section. An active control system is located on the top floor. Passive damping ratio ρ of the structure is assumed to be zero. Constraints are the same except that the first damping factor ξ_1 of the closed-loop system must be larger than 0.06. The meaning of the constraints is that under the action of normal loading cases, the structure itself resists loads and satisfies design requirements. When strong earthquake excitation acts on the structure, the active control system serves to reduce the structure's response. Objective functions are taken as structural weight W [Eq. (8)] and performance index f_6 [Eq. (15)]. The initial disturbance vector $\{z_0\}$ is assumed to be the same as the displacements caused by the application of a load of 100 kN acting on the top floor. Weighting matrices $[Q]$ and [R] of Eq. (14) are expressed as follows[21]:

$$[Q] = \begin{bmatrix} [K] & [0] \\ [0] & [M] \end{bmatrix} \tag{20}$$

and

$$[R] = [D]^T [K]^{-1} [D] \tag{21}$$

The following five design cases are investigated. In case 1, the weight of a structure is minimized without the control system. In case 2, a structural system is designed by minimizing the weight subject to structural constraints [Eq. (19)] and the control system is then developed. Cases 3–5 belong to the integrated structure/control optimization design; constraints are put on the structure [Eq. (19)] and the damping factor. Case 3 minimizes weight W, case 4 minimizes performance index f_6, and case 5 minimizes W and f_6. Using the optimization algorithm proposed in the second section, optimization results are shown in Table 2.

In case 3, which optimizes weight only, the value of the performance index is about 20 times that of case 4, in which the performance index is optimized. The weight of case 4 is 2.7 times more than that of case 3. Case 5 is a multiobjective optimization problem; the results are a compromise. Weight (2.6445) and performance index (0.2313) are situated between their best values (1.7456 and 0.0389) and worst values (4.8289 and 0.8039). Multiobjective optimization provides a basis to search for a comprehensive solution amid conflicting objectives. In case 2, the structure is designed by minimizing weight subject only to structural constraints. Then the closed-loop gain matrix $[G]$ is obtained from given structural data. In case 3, the integrated structure/control is implemented, where weight is minimized subject to structural and closed-loop constraints. Though the weight of case 3 increases 12% over case 2, the performance index decreases 35%, and the first damping factor increases almost 200%. Clearly, the integrated structure/control optimization design makes better use of resources and produces a more effective overall design. For integrated structure/control optimization design, the application of structural constraints is necessary to produce a rational and practical structure/control system.

Table 2 Optimization results of three-story frame

Case	1	2	3	4	5
Design variables					
A_1, cm^2	1020.81	1020.81	1020.33	2579.59	1492.41
A_2, cm^2	866.56	866.56	888.55	2579.57	1389.51
A_3, cm^2	612.94	612.94	888.54	2579.56	1356.78
Damping factors					
ξ_1	0	0.02138	0.06000	0.06660	0.06000
ξ_2	0	0.16528	0.17046	0.17506	0.17494
ξ_3	0	0.53495	0.53191	0.52965	0.53025
Objectives					
W^a	1.5605	1.5605	1.7456	4.8289	2.6445
f_6	#	1.2316	0.8039	0.0389	0.2313

[a]Weight W includes only column weight.

Conclusions

1) A multiobjective optimization algorithm is presented. This algorithm can directly pick the best compromise solution from the Pareto optimal set, and the solution represents the maximum possible benefit of each objective in a cooperative multiobjective optimization procedure.

2) Since most real-world structures are multimodal, single objective optimization usually cannot yield an actual optimization structural design. Through the analysis procedure, it is clear that multiobjective formulations enhance the engineering design process when several conflicting objectives must be satisfied.

3) Integrated structure/control optimization design produces more effective overall design with its ability to use available design resources and achieve ideal control at minimal cost.

4) To produce a rational and practical structure/control system, structural requirements should be included in the constraints of the optimization design procedure for the integrated structure/control system.

Appendix: Input Energy for a Multiple-Degree-of-Freedom System

When a building structure is subjected to earthquake excitations and begins to vibrate, its energy equilibrium equation based on structural dynamics theory is

$$E_K + E_D + E_A = E_I \tag{A1}$$

where E_K is kinetic energy, E_D is damping energy, E_A is absorbed energy, and E_I is input energy of the structural system. Input energy may be expressed as

$$E_I = \int_0^t \left(\sum_{i=1}^{n} m_i \dot{x}_i \ddot{x}_g \right) \mathrm{d}t \tag{A2}$$

where $\ddot{x}_g$ is base acceleration and m_i and $\dot{x}_i$ are mass and velocity relative to the base of the ith story, respectively.

Deformations of a structure are related to structural mode shapes at any moment. At time t, displacement of the structure can be expressed by the linear combination of modes of the structure

$$\{x\} = [X]\{\eta(t)\} \tag{A3}$$

where $[X]$ is the structure's mode matrix and $\{\eta(t)\}$ is the mode effective parameter vector that may be expressed as

$$\eta_i = \frac{\{x\}^T \{m\}}{m_i^*} y_i \tag{A4}$$

where

$$y_i = \frac{1}{\omega_i}\int_0^t \ddot{x}_g e^{-\xi_i\omega_i(t-\tau)} \sin\omega_i(t-\tau)\,\mathrm{d}\tau \tag{A5}$$

and

$$\{x_i\}^T = \{x_{1i}, x_{2i}, \ldots, x_{ni}\} \tag{A6}$$

$$\{m\}^T = \{m_1, m_2, \ldots, m_n\} \tag{A7}$$

$$m_i^* = \sum_{j=1}^{n} x_{ji}^2 m_j \tag{A8}$$

Substituting Eqs. (A3) and (A4) into Eq. (A2), the input energy becomes

$$E_I = \int_0^t \ddot{x}_g \sum_{i=1}^{n} m_i\dot{x}_i\,\mathrm{d}t = \sum_{i=1}^{n}\{x_i\}^T\{m\}\int_0^t \ddot{x}_g\dot{\eta}\,\mathrm{d}t$$

$$= \sum_{i=1}^{n}\frac{\left[\{x_i\}^T\{m\}\right]^2}{Mm_i^*}\int_0^t M\ddot{x}_g\dot{y}_i\,\mathrm{d}t \tag{A9}$$

where

$$M = \sum_{i=1}^{n} m_i \tag{A10}$$

For a single-degree-of-freedom system with mass M, circular frequency ω_i, damping ratio ξ_i, and input ground motion $\ddot{x}_g$, the system's input energy is[26]

$$E_{Ii} = \int_0^t M\ddot{x}_g\dot{x}_i\,\mathrm{d}t \tag{A11}$$

where

$$x_i = \frac{1}{\omega_i}\int_0^t e^{-\xi_i\omega_i(t-\tau)}\sin\omega_i(t-\tau)\,\mathrm{d}\tau \tag{A12}$$

Comparing Eqs. (A9) and (A11), the input energy of a multiple-degree-of-freedom system can be expressed as

$$E_I = \sum_{i=1}^{n}\frac{\left[\{x_i\}^T\{m_i\}\right]^2}{Mm_i^*}E_{Ii} \tag{A13}$$

Normalizing $\{x_i\}$ by

$$\{\phi_i\} = \{x_i\}\big/\sqrt{m_i^*} \tag{A14}$$

Eq. (A13) can be rewritten as

$$E_I = \sum_{i=1}^{n}\left[\frac{\{\phi_i\}^T\{m_i\}}{\sqrt{M}}\right]^2 = \sum_{i=1}^{n} D_i^2 E_{Ii} \tag{A15}$$

where D_i is the energy effective parameter of the ith mode:

$$D_i = \frac{\{\phi_i\}^T\{m_i\}}{\sqrt{M}} \tag{A16}$$

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